Quantitative Methods

Cheat Sheets

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Quantitative Methods

TIME VALUE OF MONEY

Effective Annual Rate (EAR)	**Effective annual rate =** $\left(1 + \frac{\text{Stated annual rate}}{m}\right)^m - 1$	
Single Cash Flow (Simplified formula)	$FV_N = PV \times (1 + r)^N$ $PV = \frac{FV_N}{(1 + r)^N}$	**r =** Interest rate per period **PV =** Present value of the investment **FV_N =** Future value of the investment N periods from today
Investments paying interest more than once a year	$FV_N = PV \times \left(1 + \frac{r_s}{m}\right)^{mN}$ $PV = \frac{FV_N}{\left(1 + \frac{r_s}{m}\right)^{mN}}$	**r_s =** Stated annual interest rate **m =** Number of compounding periods per year **N =** Number of years
Future Value (FV) of an Investment with Continuous Compounding	$FV_N = PVe^{r_s N}$	
Ordinary Annuity	$FV_N = A \times \left[\frac{(1 + r)^N - 1}{r}\right]$ $PV = A \times \left[\frac{1 - \frac{1}{(1 + r)^N}}{r}\right]$	**N =** Number of time periods **A =** Annuity amount **r =** Interest rate per period
Annuity Due	$FV\ A_{Due} = FV\ A_{Ordinary} \times (1 + r) = A \times \left[\frac{(1 + r)^N - 1}{r}\right] \times (1 + r)$ $PV\ A_{Due} = FV\ A_{Ordinary} \times (1 + r) = A \times \left[\frac{1 - \frac{1}{(1 + r)^N}}{r}\right] \times (1 + r)$	**A =** Annuity amount **r =** The interest rate per period corresponding to the frequency of annuity paments (for example, annual, quarterly, or monthly) **N =** Number of annuity payments

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TIME VALUE OF MONEY

Present Value (PV) of a Perpetuity	$PV_{Perpetuity} = \frac{A}{r}$	**A** = Annuity amount
Future value (FV) of a series of unequal cash flows	$FV_N = \text{Cash flow}_1(1+r)^1 + \text{Cash flow}_2(1+r)^2 \dots \text{Cash flow}_N(1+r)^N$	
Net Present Value (NPV)	$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$	**CF_t** = Expected net cash flow at time t **N** = Investment's projected life **r** = Discount rate or opportunity cost of capital
Internal Rate of Return (IRR)	$NPV = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \dots + \frac{CF_N}{(1+IRR)^N} = 0$	
Holding Period Return (HPR) No cash flows	$HPR = \frac{\text{Ending value - Beginning value}}{\text{Beginning value}}$	
Holding Period Return (HPR) Cash flows occur at the end of the period	$HPR = \frac{\text{Ending value} - \text{Beginning value} + \text{Cash flow received}}{\text{Beginning value}} = \frac{P_1 - P_0 + D_1}{\text{Beginning value}}$	**P_1** = Ending Value **P_0** = Beginning Value **D** = Cash flow/dividend received
Yield on a Bank Discount Basis (BDY)	$r_{BD} = \frac{D}{F} \times \frac{360}{t}$	**r_{BD}** = Annualized yield on a bank discount basis **D** = Dollar discount, which is equal to the difference between the face value of the bill (F) and its purchase price (P_0) **F** = Face value of the T-bill **t** = Actual number of days remaining to maturity
Effective annual yield (EAY)	$EAY = (1 + HPR)^{\frac{360}{t}} - 1$	**t** = Time until maturity **HPR** = Holding Period Return
Money market yield (CD equivalent yield)	$\text{Money market yield} = HPR \times \left(\frac{360}{t}\right) = \frac{360 \times r_{BankDiscount}}{360 - (t \times r_{BankDiscount})}$	

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STATISTICAL CONCEPTS AND MARKET RETURNS

Interval Width	**Interval Width** $= \frac{\text{Range}}{k}$	**Range =** Largest observation number - Smallest Observation or number **k =** Number of desired intervals
Relative Frequency Formula	**Relative frequency** $= \frac{\text{Interval frequency}}{\text{Observations in data set}}$	
Population Mean	$\mu = \frac{\sum_{i=1 \dots n}^{N} x_i}{N} = \frac{x_1 + x_2 + x_3 + \dots + x_N}{N}$	**N =** Number of observations in the entire population X_i **=** the i^{th} observation
Sample Mean	$\bar{x} = \frac{\sum_{i=1 \dots n}^{n} x_i}{n} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$	
Geometric Mean	$G = \sqrt[n]{X_1 X_2 X_3 \dots X_n}$	**n =** Number of observations
Harmonic Mean	$\bar{x}_n = \frac{n}{\sum_{i=1 \dots n}^{n} \left(\frac{1}{x_i}\right)}$	
Median for odd numbers	**Median** $= \left\{\frac{(n+1)}{2}\right\}$	
Median of even numbers	**Median** $= \left\{\frac{(n+2)}{2}\right\}$ **Median** $= \frac{n}{2}$	

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STATISTICAL CONCEPTS AND MARKET RETURNS

Weighted Mean	$\bar{X}_w = \sum_{i=1 \dots n}^{n} w_i X_i$	**w =** Weights **X =** Observations **Sum of all weights =** 1
Portfolio Rate of Return	$r_p = w_a r_a + w_b r_b + w_c r_c + \dots + w_n r_n$	**w =** Weights **r =** Returns
Position of the Observation at a Given Percentile y	$L_y = \left\{(n+1)\frac{y}{100}\right\}$	**y =** The percentage point at which we are dividing the distribution **L_y =** The location (L) of the percentile (Py) in the array sorted in ascending order
Range	**Range =** Maximum value - Minimum value	
Mean Absolute Deviation	$MAD = \frac{\sum_{i=1 \dots n}^{n} \lvert X_i - \bar{X} \rvert}{n}$	**X =** The sample mean **n =** Number of observations in the sample
Population Variance	$\sigma^2 = \frac{\sum_{i=1 \dots n}^{N} (X_i - \mu)^2}{N}$	**μ =** Population mean **N =** Size of the population
Population Standard Deviation	$\sigma = \sqrt{\frac{\sum_{i=1 \dots n}^{N} (X_i - \mu)^2}{N}}$	**μ =** Population mean **N =** Size of the population
Sample Variance	$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$	**X =** Sample mean **n =** Number of observations in the sample

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STATISTICAL CONCEPTS AND MARKET RETURNS

Sample Standard Deviation	$s = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}}$	**X** = Sample mean **n** = Number of observations in the sample
Semi-Variance	**Semi-variance** $= \dfrac{1}{n}\sum_{r_t < Mean}^{n}(Mean - r_t)^2$	**n** = Total number of observations below the mean r_t = Observed value
Chebyshev Inequality	Percentage of observations within *k* standard deviations of the arithmetic mean $> 1 - \dfrac{1}{k^2}$	**k** = Number of standard deviations from the mean
Coefficient of Variation	$CV = \dfrac{s}{\bar{x}}$	**s** = Sample standard deviation $\bar{x}$ = Sample mean
Sharpe Ratio	**Sharpe Ratio** $= \dfrac{R_p - R_f}{\sigma_p}$	R_p = Mean return to the portfolio R_F = Mean return to a risk-free asset σ_p = Standard deviation of return on the portfolio
Skewness	$S_K = \left[\dfrac{n}{(n-1)(n-2)}\right] \times \dfrac{\sum_{i=1 \ldots n}^{n}(x_i - \bar{x})^3}{s^3}$	**n** = Number of observations in the sample **s** = Sample standard deviation
Kurtosis	$K_E = \left[\dfrac{n(n+1)}{(n-1)(n-2)(n-3)} \times \dfrac{\sum_{i=1 \ldots n}^{n}(x_i - \bar{x})^4}{S^4}\right] - \dfrac{3(n-1)^2}{(n-2)(n-3)}$	**n** = Sample size **s** = Sample standard deviation

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PROBABILITY CONCEPTS

Odds FOR E	**Odds FOR E** $= \frac{P(E)}{1 - P(E)}$	**E =** Odds for event **P(E) =** Probability of event
Conditional Probability	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$	where **P(B)** ≠ 0
Additive Law (The Addition Rule)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
The Multiplication Rule (Joint Probability)	$P(A \cap B) = P(A\|B) \times P(B)$	
The Total Probability Rule	$P(A) = P(A\|S_1) \times P(S_1) + P(A\|S_2) \times P(S_2) + \ldots + P(A\|S_n) \times P(S_n)$	**S1, S2, ..., Sn** are mutually exclusive and exhaustive scenarios or events
Expected Value	$E(X) = P(A)X_A + P(B)X_B + \ldots + P(n)X_n$	**P(n) =** Probability of an variable X_n **=** Value of the variable
Covariance	$COV_{xy} = \frac{(x - \overline{x})(y - \overline{y})}{n - 1}$	**x =** Value of x $\overline{X}$ **=** Mean of x values **y =** Value of y $\overline{y}$ **=** Means of y **n =** Total number of values
Correlation	$\rho = \frac{cov_{xy}}{\sigma_x \sigma_y}$	σ_x **=** Standard Deviation of x σ_y **=** Standard Deviation of y COV_{xy} **=** Covariance of x and y
Variance of a Random Variable	$\sigma^2 X = \sum_{i=1 \ldots n}^{n} (x - E(x))^2 \times P(x)$	The sum is taken over all values of **x** for which **p(x) > 0**
Portfolio Expected Return	$E(R_P) = E(w_1 r_1 + w_2 r_2 + w_3 r_3 + \ldots + w_n r_n)$	**w =** Constant **r =** Random variable
Portfolio Variance	$Var(R_p) = E[(R_p - E(R_p))^2] = [w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2w_1 w_2 Cov(R_1 R_2) + 2w_2 w_3 Cov(R_2 R_3) + 2w_1 w_3 Cov(R_1 R_3)]$	R_p **=** Return on Portfolio
Bayes' Formula	$P(A\|B) = \frac{P(B\|A) \times P(A)}{P(B)}$	
The Combination Formula	$nCr = \binom{n}{c} = \frac{n!}{(n - r)!\, r!}$	**n =** Total objects **r =** Selected objects
The Permutation Formula	$nPr = \frac{n!}{(n - r)!}$	

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COMMON PROBABILITY DISTRIBUTIONS

The Binomial Probability Formula	$P(x) = \frac{n!}{(n-x)!\,x!} p^x \times (1-p)^{n-x}$	**n** = Number of trials **x** = Up moves p^x = Probability of up moves $(1-p)^{n-x}$ = Probability of down moves
Binomial Random Variable	**E(X)** = np **Variance** = np(1 - p)	**n** = Number of trials **p** = Probability
For a Random Normal Variable X	90% confidence interval for X is $\bar{x} - 1.65s; \bar{x} + 1.65s$ 95% confidence interval for X is $\bar{x} - 1.96s; \bar{x} + 1.96s$ 99% confidence interval for X is $\bar{x} - 2.58s; \bar{x} + 2.58s$	**s** = Standard error **1.65** = Reliability factor $\bar{x}$ = Point estimate
Safety-First Ratio	$SF_{Ratio} = \left[\frac{E(R_p) - R_L}{\sigma_p}\right]$	R_p = Portfolio Return R_L = Threshold level σ_p = Standard Deviation
Continuously Compounded Rate of Return	$FV = PV \times e^{i \times t}$	**i** = Interest rate **t** = Time **ln e** = 1 **e** = The exponential function, equal to 2.71828

SAMPLING AND ESTIMATION

Sampling Error of the Mean	Sample Mean - Population Mean	
Standard Error of the Sample Mean (Known Population Variance)	$SE = \frac{\sigma}{\sqrt{n}}$	**n** = Number of samples **σ** = Standard deviation
Standard Error of the Sample Mean (Unknown Population Variance)	$SE = \frac{S}{\sqrt{n}}$	**s** = Standard deviation in unknown population's sample
Z-score	$Z = \frac{x - \mu}{\sigma}$	**x** = Observed value **σ** = Standard deviation **μ** = Population mean
Confidence Interval for Population Mean with z	$\bar{x} - Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}; \bar{x} + Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$	$Z_{\alpha/2}$ = Reliability factor $\bar{x}$ = Mean of sample **σ** = Standard deviation **n** = Number of trials/size of the sample
Confidence Interval for Population Mean with t	$\bar{x} - t_{\frac{\alpha}{2}} \times \frac{s}{\sqrt{n}}; \bar{x} + t_{\frac{\alpha}{2}} \times \frac{s}{\sqrt{n}}$	$t_{\alpha/2}$ = Reliability factor **n** = Size of the sample **s** = Standard deviation
z or t-statistic?	**Z** ⟶ known population, standard deviation σ, no matter the sample size **t** ⟶ unknown population, standard deviation s, and sample size below 30 **Z** ⟶ unknown population, standard deviation s, and sample size above 30	

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HYPOTHESIS TESTING

Test Statistics: Population Mean	$z_\alpha = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$; $t_{n-1,\alpha} = \dfrac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$	t_{n-1} = t-statistic with n − 1 degrees of freedom *(n is the sample size)* $\overline{x}$ = Sample mean **μ** = Hypothesized value of the population mean **s** = Sample standard deviation
Test Statistics: Difference in Means - Sample Variances Assumed Equal (Independent samples)	**t-statistic** $= \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{\frac{1}{2}}}$ $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	**Number of degrees of freedom =** $= n_1 + n_2 - 2$
Test Statistics: Difference in Means - Sample Variances Assumed Unequal (Independent samples)	**t-statistic** $= \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{\frac{1}{2}}}$ **degrees of freedom** $= \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2}}$	**S** = Standard deviation of respective sample **n** = Total number of observations in the respective population
Test Statistics: Difference in Means - Paired Comparisons Test (Dependent samples)	$t = \dfrac{\overline{d} - \mu_{d0}}{S_d}$, where $\overline{d} = \dfrac{1}{n}\sum_{i=1 \dots n}^{n} d_i$	**degrees of freedom** = n - 1 **n** = Number of paired observations **d** = Sample mean difference S_d = Standard error of d
Test Statistics: Variance Chi-square Test	$\chi^2_{n-1} = \dfrac{(n-1)s^2}{\sigma_0^2}$	**degrees of freedom** = n - 1 s^2 = Sample variance σ_0^2 = Hypothesized variance
Test Statistics: Variance F-Test	$F = \dfrac{s_1^2}{s_2^2}$, where $s_1^2 > s_2^2$	**degrees of freedom** = $n_1 - 1$ and $n_2 - 1$ s_1^2 = Larger sample variance s_2^2 = Smaller sample variance

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